Laplace Tranform

- Definition : If f(t) is a function of t satisfying certain conditions , then the definite integral

$$\varphi(s) = \int_0^\infty e^{-st} . f(t) dt \; ; s > 0$$

if this exists then it is called as Laplace transform of f(t) and is denoted as

$$L[f(t)] = \varphi(s) = \int_0^\infty e^{-st} . f(t) dt \; ; s > 0 \; .$$

Examples

- Example : 1 Find the Laplace transform of f(t), Where

  f(t) = t , for 0 < t < 4 and f(t) = 5 , for t > 4.

- Solution : By Definition ,

$$L[f(t)] = \varphi(s) = \int_0^\infty e^{-st} . f(t) dt \; ; s > 0$$

therefore

$$L[f(t)] = \int_0^4 e^{-st} .t dt + \int_4^\infty e^{-st} .5 \, dt$$

Solution

$$= \left[(t)\left(\frac{e^{-st}}{-s}\right) - \left(\frac{e^{-st}}{s^2}\right)\right] \text{from 0 to 4}$$

$$+ 5\left[\frac{e^{-st}}{-s}\right] \textit{from } 4 \textit{ to } \infty$$

$$= \left[\frac{-4}{s}e^{-4s} - \frac{1}{s^2}e^{-4s} + \frac{1}{s^2} + \frac{5}{s}e^{-4s}\right]$$

$$= \frac{1}{s^2} + \left(\frac{1}{s} - \frac{1}{s^2}\right).e^{-4s}$$

Laplace transform of standard functions

- 1) L[e^{at}] = $\frac{1}{s-a}$
- 2) L$[sinat] = \frac{a}{s^2+a^2}$
- 3) L$[cosat] = \frac{s}{s^2+a^2}$
- 4) L$[coshat] = \frac{s}{s^2-a^2}$

- 5) L$[sinhat] = \dfrac{a}{s^2 - a^2}$

- 6) L$[1] = \dfrac{1}{s}$

- 7) L$[t^n] = \dfrac{n!}{s^n}$