Laplace Tranform

• Definition : If f(t) is a function of t satisfying certain conditions , then the definite integral

$$\varphi(s) = \int_0^\infty e^{-st} \cdot f(t) dt ; s > 0$$

if this exists then it is called as Laplace transform of f(t) and is denoted as $L[f(t)] = \varphi(s) = \int_{0}^{\infty} e^{-st} f(t) dt; s > 0.$

Examples

• Example : 1 Find the Laplace transform of f(t), Where

f(t) = t, for 0 < t < 4 and f(t) = 5, for t > 4.

• Solution : By Definition ,

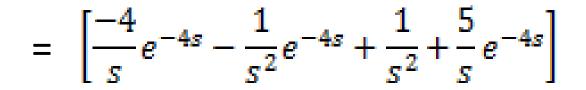
$$L[f(t)] = \varphi(s) = \int_0^\infty e^{-st} f(t)dt ; s > 0$$

therefore
$$L[f(t)] = \int_0^4 e^{-st} dt + \int_4^\infty e^{-st} dt$$

Solution

$$= \left[(t) \left(\frac{e^{-st}}{-s} \right) - \left(\frac{e^{-st}}{s^2} \right) \right]$$
from 0 to 4

+
$$5\left[\frac{e^{-st}}{-s}\right]$$
 from 4 to ∞



$$= \frac{1}{s^2} + \left(\frac{1}{s} - \frac{1}{s^2}\right) \cdot e^{-4s}$$

Laplace transform of standard functions

• 1) L[
$$e^{at}$$
] = $\frac{1}{s-a}$

• 2)
$$L[sinat] = \frac{a}{s^2 + a^2}$$

• 3)
$$L[cosat] = \frac{s}{s^2 + a^2}$$

• 4)
$$L[coshat] = \frac{s}{s^2 - a^2}$$

- 5) $L[sinhat] = \frac{a}{s^2 a^2}$
- 6) $L[1] = \frac{1}{s}$
- 7) $L[t^n] = \frac{n!}{s^n}$